

USN

--	--	--	--	--	--	--	--	--	--

15MAT11

## First Semester B.E. Degree Examination, Jan./Feb. 2021 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 80

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

- 1 a. Find the  $n^{\text{th}}$  derivative of  $x^2 e^x \cos x$ . (06 Marks)
- b. Find the angle ( $\phi$ ) between the radius vector and tangent of the curve  $r = a(1 + \sin \theta)$ . Also determine the slope of the curve  $a + \theta = \frac{\pi}{2}$ . (05 Marks)
- c. Obtain the angle of intersection of the polar curves  $r = a \log \theta$ ;  $r = \frac{a}{\log \theta}$ . (05 Marks)

### OR

- 2 a. If  $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$ , then prove that  $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$ . (06 Marks)
- b. Find the pedal equation of the polar curve  $r^n = a^n \cos n\theta$ . (05 Marks)
- c. Find the radius of curvature at any point 't' of the curve,  $x = a \left( \cos t + \log \tan \frac{t}{2} \right)$ ,  $y = a \sin t$ . (05 Marks)

### Module-2

- 3 a. Expand  $y = \log x$  in powers of  $(x - 1)$  upto fourth degree term and hence evaluate  $\log(1.1)$ . (06 Marks)
- b. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{\frac{1}{x^2}}$ . (05 Marks)
- c. If  $u = \log(x^3 + y^3 + z^3 - 3xy)$ , prove the following:
  - (i)  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x + y + z}$
  - (ii)  $\left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x + y + z)^2}$  (05 Marks)

### OR

- 4 a. Prove that, using MaClaurin's series,  $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \dots$  (06 Marks)
- b. If  $u = \cot^{-1} \left( \frac{x + y}{\sqrt{x} + \sqrt{y}} \right)$ , prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{4} \sin 2u$  (05 Marks)
- c. If  $u = \frac{xy}{z}$ ,  $v = \frac{yz}{x}$ ,  $w = \frac{zx}{y}$ , find  $J \left( \frac{u, v, w}{x, y, z} \right)$ . (05 Marks)

**Module-3**

- 5 a. Find the constants 'a' and 'b' such that  $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$  is irrotational. Also find a scalar potential  $\phi$  such that  $\vec{F} = \nabla\phi$ . (06 Marks)
- b. Find the directional derivative of  $\phi = x^2yz + 4xz^2$  at the point (1, -2, -1) along the vector  $\hat{A} = 2\hat{i} - \hat{j} - 2\hat{k}$ . (05 Marks)
- c. A particle moves along the curve  $\vec{r} = 2t\hat{i} + (t^2 - 4t)\hat{j} + (3t - 5)\hat{k}$ . Find the components of velocity and acceleration in the direction of the vector  $\vec{A} = \hat{i} - 3\hat{j} + 2\hat{k}$  at  $t = 2$ . (05 Marks)

OR

- 6 a. For any scalar field  $\phi$  and any vector field  $\vec{A}$ , prove that  $\nabla \times (\phi\vec{A}) = \phi(\nabla \times \vec{A}) + (\nabla\phi) \times \vec{A}$ . (06 Marks)
- b. If  $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$ , find  $\text{div}(\vec{F})$  and  $\text{curl}(\vec{F})$  at the point (1, 2, 3). (05 Marks)
- c. Find the angle between the tangents to the curve  $x = t^2 + 1$ ,  $y = 4t - 3$ ,  $z = 2t^2 - 6t$  at  $t = 1$  and  $t = 2$ . (05 Marks)

**Module-4**

- 7 a. Obtain the reduction formula for  $\int_0^{\pi/2} \cos^n x dx$ . (06 Marks)
- b. Solve  $\frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0$  (05 Marks)
- c. Find the orthogonal trajectories of the family of ellipses  $\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1$ . (05 Marks)

OR

- 8 a. Evaluate  $\int_0^a x\sqrt{ax - x^2} dx$ . (06 Marks)
- b. Solve  $\frac{dy}{dx} + \frac{y}{x} = y^2x$ . (05 Marks)
- c. The temperature of a body drops from  $100^\circ\text{C}$  to  $75^\circ\text{C}$  in 10 minutes when the surrounding air is at  $20^\circ\text{C}$ . What will be its temperature after half an hour? When will the temperature be  $25^\circ\text{C}$ ? (05 Marks)

**Module-5**

- 9 a. Show that the linear transformation :  $y_1 = 2x_1 + x_2 + x_3$ ;  $y_2 = x_1 + x_2 + 2x_3$ ;  $y_3 = x_1 - 2x_3$  is regular. Also, determine the inverse transformation. (06 Marks)
- b. Find the dominant eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

using Rayleigh's power method. Choose  $[1, 0, 0]^T$  as the initial vector and perform five iterations. (05 Marks)



15MAT11

- c. Solve the following system of equation by Gauss elimination method:

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

(05 Marks)

OR

- 10 a. Employ the Gauss-Seidal method to solve the following system:

$$9x - y + 2z = 9$$

$$x + 10y - 2z = 15$$

$$-2x + 2y + 13z = 17$$

Choose (1, 1, 1) as the starting solution and carry out four iterations.

(06 Marks)

- b. Reduce the following matrix to diagonal form:

$$A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$$

(05 Marks)

- c. Obtain the canonical form of the quadratic form  $3x^2 + 2y^2 - z^2 + 12yz + 8zx - 4xy$ .

(05 Marks)

\*\*\*\*\*